Problem Set # 2 (Assigned 7 September, Due 15 September)

1. Simplify the following expressions using the laws and theorems of Boolean Algebra:
   (a) \[ S(A,B,C) = A' B' C + A' B C' + A B' C' + A B C \]
   (b) \[ F(A,B,C) = A' B' C' + A' B' C + A B' C' + A B C' + A B C \]
   (c) \[ G(A,B,C,D) = A' B' C' D' + A' B' C D' + A B' C' D' + A B C' D + A B C D + A B C' D' + A B C D \]

2. Use K-maps on the expressions of Problem 1. Show your work in K-map form. For 1(a)-(c):
   (a) Find the minimized sum of products form.
   (b) Find the minimized product of sums form.
   (c) Find the minimized sum of products form of the function’s complement.
   (d) Find the minimized product of sums form of the function’s complement.

3. Consider the following multilevel Boolean expressions:
   (i) \[ F(A,B,C,D) = (A + (B C))(C' + D) \]
   (ii) \[ G(A,B,C,D) = ((A + B') D) + (A + (B C)) \]
   Perform the following:
   (a) Show how to implement each function as a multilevel NAND only gate level implementation.
   (b) Repeat (a), but using NOR gates only.
   (c) Find the two level minimized Sum of Products forms.
   (d) Find the two level minimized Products of Sums forms.
   (e) Briefly compare the implementation complexities in terms of gates and literals. For each function, which achieves the “simplest” implementation.

4. Develop a minimized Boolean implementation of a 2-bit combinational divider. The subsystem has two 2-bit inputs A,B and C,D, and generates two 2-bit outputs, the quotient W,X, and the remainder Y,Z.
   (a) Draw the truth tables for W,X(A,B,C,D) and Y,Z(A,B,C,D).
   (b) Minimize the functions W, X, Y, Z using 4-variable K-maps. Write down the Boolean expressions for the minimized Sum of Products form of each function.
   (c) Repeat the minimization process, this time deriving Product of Sums form.

5. Consider a combinational logic subsystem that determines if a 4-bit binary quantity A,B,C,D in the range of 0000 (0) through 1011 (11 in base 10) is divisible by the decimal numbers two, three, or six. That is, the function is true if the input can be divided by the indicated amount with no remainder (e.g., BY2(0110), BY3(0110), and BY6(0110) are all true). Assume that the binary patterns 1100 (12) through 1111 (15) are “don’t cares.”
   (a) Draw the truth tables for BY2(A,B,C,D), BY3(A,B,C,D), BY6(A,B,C,D).
   (b) Minimize the functions using 4-variable K-maps to derive minimized Sum of Products forms.
   (c) Can you further simplify the result by using a multilevel logic implementation? If so, how?